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Author: Frank Regier, OD5CG

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Series-Section Transmission-Line Impedance Matching

Nearly everyone who's worked with antennas knows about stub matching. But series-section matching has a number of advantages over stub tuning. Here's how it's done.

By Frank A. Regier,* OD5CG

Series-section matching may be a strange term to you, but the principle is probably a familiar one. Let's say you have put together a 35-Ω antenna system, perhaps an array of elements for a repeater, and you want to feed it with 75-Ω hardline. How would you make the impedance transformation from 35 to 75 ohms? There are a number of ways, of course — an rf impedance transformer and stub matching, to name two. But in this case perhaps the simplest would be to use a quarter-wavelength line transformer at the antenna. The impedance required for the matching-line section may be calculated from the equation

$$Z1 = \sqrt{Z_{load} \times Z_{line}}$$

where Z1 is the impedance needed for the 1/4-λ matching section and Z_{load} is the purely resistive impedance to be matched to Z_{line}. In this example the value for Z1 conveniently works out to be 51.2 Ω, and a line having a nominal impedance of 50 to 53 ohms may be used. But what happens when the load is not purely resistive, or when the required impedance for Z1 is some uncommon value? In these cases it may be necessary to use another form of matching.

This article introduces a new impedance-matching system called the series-section transformer. It has worthwhile advantages over either stub tuning or the 1/4-λ transformer. The series-section transformer is illustrated in Fig. 1 and bears considerable resemblance to the 1/4-λ transformer. (Actually the 1/4-λ transformer is a special case of the series-section transformer.) The important differences are, first, that the matching section may not be located exactly at the load, second, that it may be less than a quarter wavelength long, and third and

*Dept. of Electrical Engineering, American University of Beirut, Beirut, Lebanon

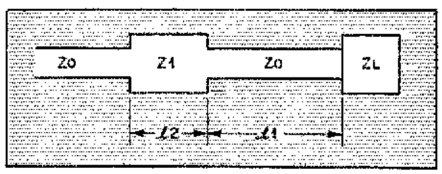


Fig. 1 — Series-section transformer, Z1, for matching transmission-line Zo to load, Z_L.

most important, that there is great freedom in the choice of the characteristic impedance of the matching section.

In fact, the matching section can have *any* characteristic impedance that is not too close to that of the main line. Because of this freedom, it is almost always possible to find a length of commercially available line that will be suitable as a matching section. As an example, consider a 75-Ω line, a 300-Ω matching section, and a pure-resistance load (example only; complex loads can also be matched). It can be shown that such a section may be used to match *any* resistance between 5 Ω and 1200 Ω to the main line.

The design of a series-section transformer consists of determining the length $\ell 2$ of the series or matching section and the distance $\ell 1$ from the load to the point where the section should be inserted into the main line. Three quantities must be known. These are the characteristic impedances of the main line and of the matching section, both assumed purely resistive, and the complex-load impedance. Either of two design methods may be used. One is algebraic, and the other is a graphical method using the Smith Chart. You can take your choice.

Algebraic Design Method

The two lengths $\ell 1$ and $\ell 2$ are to be determined from the characteristic impedances of the main line and the matching section, Zo and Z1 respectively, and the load impedance $Z_L = R_L + jX_L$. The derivation may be found elsewhere.[1,2] Only the essential results are presented here.

The first step is to determine the normalized impedances.

$$n = \frac{Z1}{Zo} \quad \text{(Eq. 1a)}$$

$$r = \frac{R_L}{Zo} \quad \text{(Eq. 1b)}$$

$$x = \frac{X_L}{Zo} \quad \text{(Eq. 1c)}$$

Next, $\ell 2$ and $\ell 1$ are determined from the relations

$$\tan \ell 2 = B = \pm \sqrt{\frac{(r-1)^2 + x^2}{r\left(n - \frac{1}{n}\right)^2 - (r-1)^2 - x^2}} \quad \text{(Eq. 2)}$$

$$\tan \ell 1 = A = \frac{\left(n - \frac{r}{n}\right)B + x}{r + xnB - 1} \quad \text{(Eq. 3)}$$

Lengths $\ell 2$ and $\ell 1$ thus determined are electrical lengths in degrees. Actual lengths are obtained by dividing by 360° and multiplying by the wavelength measured along the line (main line or matching section, as the case may be), taking the velocity factor of the line into account.

In Eq. 2 the sign of B may be chosen either positive or negative, but the positive sign is preferred because it results in a shorter matching section. In Eq. 3 the sign of A may not be chosen but can turn out to be either positive or negative. If a negative sign occurs and an electronic calculator is then used to determine $\ell 1$, a negative electrical length will result. If this happens, add 180°. The resultant electrical length will be correct both physically and mathematically.

[1]References appear on page 16.

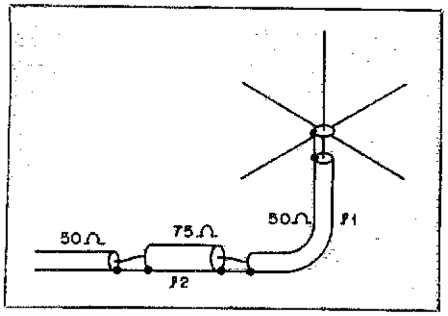


Fig. 2 — Example of series-section matching. A 38-Ω antenna is matched to 50-Ω coax by means of a length of 75-Ω cable.

In calculating B from Eq. 2, it can happen that the quantity under the radical is negative, leading to an imaginary value for B. This would mean that Z1, the impedance of the matching section, is too close to Zo and should be changed.

Limits on the characteristic impedance of Z1 may be calculated in terms of the standing-wave ratio produced by the load on the main line without matching. For matching to occur, Z1 should either be greater than $Zo\sqrt{SWR}$ or less than $Zo/\sqrt{SWR}$.

An Example

As an example, suppose we want to feed a 29-MHz groundplane vertical antenna with RG-58-type foam-dielectric coax (Fig. 2). We'll assume the antenna impedance to be 38 ohms, pure resistance, and use a length of RG-59/U foam-dielectric coax as the series section.

Zo is 50 ohms, Z1 is 75 ohms, and both cables have a velocity factor of 0.79. (From above, Z1 must have an impedance greater than 57.4 Ω or less than 43.6 Ω.) The design steps are as follows.

From Eqs. 1a through 1c,
$n = 1.5$, $r = 0.76$, and $x = 0$.

From Eq. 2, $B = 0.3500$ (positive sign chosen) and $\ell 2 = 19.29°$.

From Eq. 3, $A = -1.4486$. Calculating $\ell 1$ yields $-55.38°$. Adding 180° to obtain a positive result gives $\ell 1 = 124.62°$.

To find the physical lengths $\ell 1'$ and $\ell 2'$ we first find the free-space wavelength.

$$\lambda o = \frac{984}{f_{MHz}} \text{ feet}$$

and the transmission-line wavelength

$$\lambda = \lambda o \times \text{velocity factor}$$

In the present case we find $\lambda = 26.81$ ft. Finally we have

$$\ell 1' = \frac{\ell 1 \times \lambda}{360} = 9.28 \text{ ft, and}$$

$$\ell 2' = \frac{\ell 2 \times \lambda}{360} = 1.44 \text{ ft}$$

This completes the calculations. Construction consists of cutting the main coax at a point 9.28 ft from the antenna and adding a 1.44-ft length of the 75-Ω cable.

The Quarter-Wave Transformer

The antenna in the preceding example could have been matched by a 1/4-λ transformer at the load. Such a transformer would have a characteristic impedance of 43.6 Ω (from the equation at the beginning of this article). It is interesting to see what happens in the design of a series-section transformer if this value is chosen as the characteristic impedance of the series section.

Following the same steps as before, we find $n = 0.872$, $r = 0.76$, and $x = 0$.

From these values and Eq. 2 we find $B = \infty$ and $\ell 2 = 90°$. Further, $A = 0$ and $\ell 1 = 0°$. These results represent a quarter-wave section at the load, and indicate that, as stated earlier, the quarter-wave transformer is indeed a special case of the series-section transformer.

Another Example

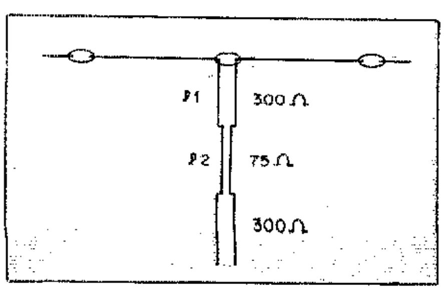
Fig. 3 shows another example, in which a series-section of 75-Ω twin lead is used to match the 75-Ω center impedance of a resonant dipole to a 300-Ω line. Following the same steps once again, we find
$n = 0.25$, $r = 0.25$, and $x = 0$.
From Eq. 2,
$B = 0.4364$ and $\ell 2 = 23.58°$
and from Eq. 3,
$A = 0.4364$ and $\ell 1 = 23.58°$.

Note that $\ell 1 = \ell 2$. This always occurs when $n = r$ and $x = 0$, and characterizes the *alternated-line transformer*,[3] used for matching two cables of different impedance (the antenna could just as well have been a 75-Ω cable), using displaced sections of the two cables being matched.

Lengths $\ell 1$ and $\ell 2$ in this case can be determined either in the usual way, or from the simplified relationship:

$$\ell 1 = \ell 2 = \tan^{-1} \sqrt{\frac{n(n-1)}{n^3 - 1}}$$

Fig. 3 — Another example of series-section matching.



Smith-Chart Solution

A series-section transformer can be designed graphically with the aid of a Smith Chart, but this requires the use of the chart in its unfamiliar off-center mode. This mode is described in the next two paragraphs.

Fig. 4 shows the Smith Chart used in its familiar centered mode, with all impedances normalized to that of the transmission line, in this case 75 ohms, and all constant-SWR circles concentric with the normalized value r = 1 at the chart center. An actual impedance is recovered by multiplying a chart reading by the normalizing impedance of 75 ohms. If the actual (unnormalized) impedances represented by a constant-SWR circle in Fig. 4 are instead divided by a normalizing impedance of 300 ohms, a different picture results. A Smith Chart shows all possible impedances, and so a closed path such as a constant-SWR circle in Fig. 4 must again be represented by a closed path. In fact, it can be shown that the path remains a circle, but that the

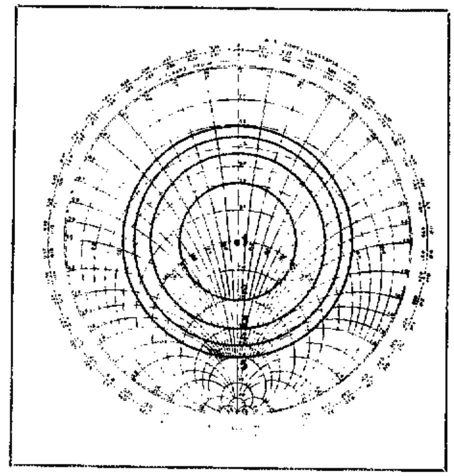


Fig. 4 — Constant-SWR circles for SWR = 2, 3, 4 and 5, showing impedance variation along 75-Ω line, normalized to 75 Ω. Actual impedance is obtained by multiplying chart reading by 75 ohms.

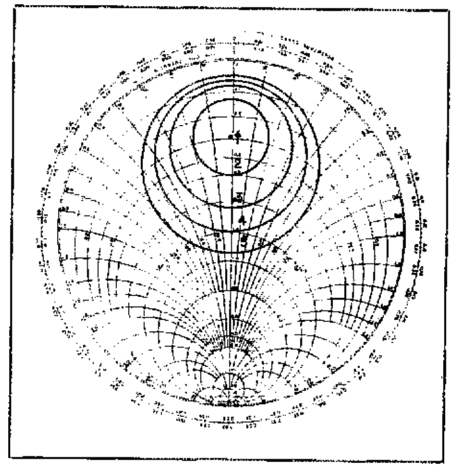


Fig. 5 — Paths of constant SWR for SWR = 2, 3, 4 and 5, showing impedance variation along 75-Ω line, normalized to 300 Ω. Normalized impedances differ from those in Fig. 4, but actual impedances are obtained by multiplying chart readings by 300 ohms and are the same as those corresponding in Fig. 4. Paths remain circles but are no longer concentric. One, the matching circle, SWR = 4 in this case, passes through the chart center and is thus the locus of all impedances which can be matched to a 300-Ω line.

constant-SWR circles are no longer concentric. Fig. 5 shows the circles which result when the impedances along a mismatched 75-Ω line are normalized by dividing by 300 ohms instead of 75. The constant-SWR circles still surround the point corresponding to the characteristic impedance of the line (r = 0.25) but are no longer concentric with it. Note that the normalized impedances read from corresponding points on Figs. 4 and 5 are different but that the actual, unnormalized, impedances are exactly the same.

Let's turn now to the example shown in Fig. 6. A complex load of Z_L = 600 + j900 ohms is to be fed with 300-Ω line, and a 75-Ω series section is to be used. These characteristic impedances agree with those used in Fig. 5, and thus Fig. 5 can be used to find the impedance variation along the 75-Ω series section. In particular, the constant-SWR circle which passes through the chart center, SWR = 4 in this case, passes through all the impedances (normalized to 300 ohms) which the 75-Ω series section is able to match to the 300-Ω main line. The length $\ell1$ of 300-Ω line has the job of transforming the load impedance to some impedance on this matching circle.

Fig. 7 shows the whole process more clearly, with all impedances normalized to 300 Ω. Here the normalized load impedance z_L = 2 + j3 is shown at R, and the matching circle appears centered on the real axis and passing through the points r = 1 and r = n^2 = 0.0625. A constant-SWR circle is drawn from R to an intersection with the matching circle at Q or Q′ and the corresponding length $\ell1$ (or $\ell1'$) can be read directly from the Smith Chart.

Although the impedance locus from Q to P is shown in Fig. 7, the length $\ell2$ cannot be determined directly from this chart. This is because the matching circle is not concentric with the chart center, as it must be if the length indications on the periphery of the Smith Chart are to be used. This problem is overcome by forming Fig. 8, which is the same as Fig. 7 except that all impedances have been divided by n = 0.25, resulting in a Smith Chart normalized to 75 ohms instead of 300. The matching circle and the chart center are now concentric, and the series-section length $\ell2$, the distance between Q and P, can be taken directly from the chart.

In fact it is not necessary to construct

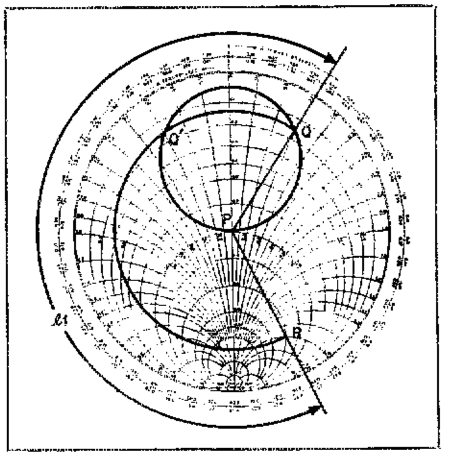


Fig. 7 — Smith Chart representation of example shown in Fig. 6. Impedance locus always has clockwise direction from load to generator, first along constant-SWR circle from load at R to intersection with matching circle at Q or Q′, then along matching circle to chart center at P. Length $\ell1$ can be determined directly from chart.

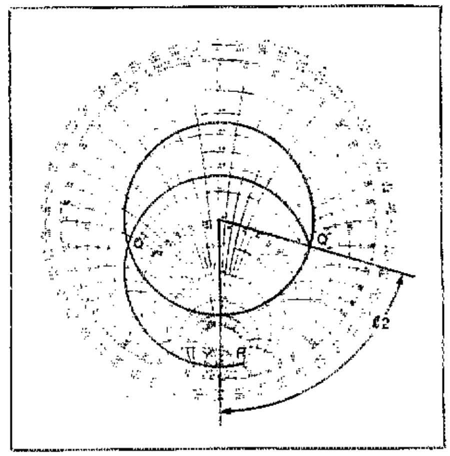


Fig. 8 — Same impedance locus as shown in Fig. 7 except normalized to 75 ohms instead of 300. The matching circle is now concentric with chart center, and $\ell2$ can be determined directly from chart. In this example, $\ell1$ = 0.332 λ and $\ell2$ = 0.102 λ.

the entire impedance locus shown in Fig. 8. It is sufficient to plot Z_Q/n (Z_Q is read from Fig. 7) and Z_P/n = 1/n, connect them by a circular arc centered on the chart center, and to determine the arc length $\ell2$ from the Smith Chart.

The steps necessary to design a series-section transformer by means of the Smith Chart can now be listed:

1) Normalize all impedances by dividing by the characteristic impedance of the main line.

2) On a Smith Chart plot the normalized load impedance z_L at R and construct the matching circle so that its center is on the real axis and it passes through the points r = 1 and r = n^2.

3) Construct a constant-SWR circle centered on the chart center through point R. This circle should intersect the matching circle at two points. One of these points, normally the one resulting in the shorter clockwise distance along the matching circle to the chart center, is chosen as point Q, and the clockwise distance from R to Q is read from the chart and taken to be $\ell1$.

4) Read the impedance Z_Q from the chart, calculate Z_Q/n and plot it as point Q on a second Smith Chart. Also plot r = 1/n as point P.

5) On this second chart construct a circular arc, centered on the chart center, clockwise from Q to P. The length of this arc, read from the chart, represents $\ell2$. The design of the transformer is now complete.

The Smith Chart construction shows that two design solutions are usually possible, corresponding to the two intersections of the load constant-SWR circle with the matching circle, and also corresponding to positive and negative values of the square-root radical in Eq. 2. It may happen, however, that the load circle misses the matching circle completely, in which case no solution is possible. The cure is to enlarge the matching circle by choosing a series section whose impedance departs more from that of the main line.

A final possibility is that, rather than intersecting the matching circle, the load circle is tangent to it. There is then but one solution — that of the 1/4-λ transformer.

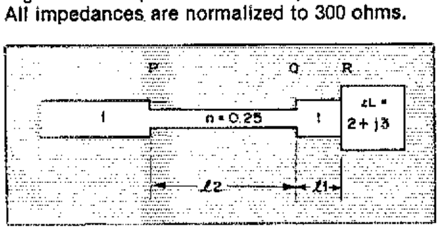
In conclusion, the series-section transformer is a convenient form of matching. It consists of a line section of not over a quarter-wavelength inserted into the main line at some point within a half-wavelength of the load. The characteristic impedance of the series section, Z1, can be either greater than or less than Zo, the characteristic impedance of the main line. The only restriction on the choice of Z1 is that it should not be too near Zo.

For a given Z1 it can be shown that a 1/4-λ matching section can handle the greatest mismatch. Lesser mismatches require shorter matching sections.

Several well-known matching arrangements turn out to be special cases of series-section matching. In addition to the quarter-wave transformer and the alternated-line transformer, it can be shown,[3] by allowing Z1 to approach zero, that stub-matching, too, is a special form of the series-section transformer.

Of the two design methods presented, the algebraic method is probably the easier if an electronic calculator is available. The Smith Chart method is a practical alternative and does provide additional insight into the operation of the series-section transformer.

Fig. 6 — Example for solution by Smith Chart. All impedances are normalized to 300 ohms.



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